Studying pion effects on the chiral phase transition

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Abstract

We investigate the chiral phase transition at finite temperatures and zero chemical potential with Dyson-Schwinger equations. Our truncation for the quark-gluon interaction includes mesonic degrees of freedom, which allows us to study the impact of the pions on the nature of the phase transition. Within the present scheme we find a five percent change of the critical temperature due to the pion backreaction whereas the mean field character of the transition is not changed.

We investigate the chiral transition of two flavor QCD in the chiral limit (i.e. massless quarks). Assuming that effective restoration of axial vector symmetry takes place at temperatures above the critical temperature of chiral symmetry restoration the transition is expected to be a second order phase transition falling in the O(4)-universality class [1]. This motivates the application of effective models constructed in terms of order parameters to describe chiral transitions, see e.g. [1, 2] and references therein. Explicit symmetry breaking changes the second order transition to a smooth crossover.

Here we study the chiral symmetry restoration with Dyson-Schwinger equations (DSE). In order to take into account the relevant degrees of freedom, i.e. the ones which retain long-range fluctuations in the vicinity of a second order phase transition, we need to employ a truncation of the quark-gluon vertex that includes mesonic fluctuations. Such a scheme has been proposed in [3]. At finite temperatures it leads to a renormalised quark DSE of the form

$$S^{-1}(\omega_n,\vec{p}) = Z_2 S_0^{-1}(\omega_n,\vec{p}) + Z_{1F}\; g^2\,\frac{4}{3}\,T\sum_{m=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}\gamma_\mu S(\omega_m,\vec{q})\Gamma_\nu(\omega_m,\vec{q};\omega_n,\vec{p})D_{\nu\mu}(\Omega_{n-m},\vec{p}-\vec{q}), \quad (1)$$

where $S^{-1}(\omega_n,\vec{p}) = (i\,\vec{\gamma}\cdot\vec{p}\,A(\omega_n,\vec{p}) + i\,\gamma_4\omega_n\,C(\omega_n,\vec{p}) + B(\omega_n),\vec{p})$ is the inverse dressed quark propagator with dressing functions $A,\ B,\ C$ and S_0^{-1} its bare counterpart. Z_2 and Z_{1F} are renormalisation constants and $\omega_n = \pi T(2n+1)$ is the fermion Matsubara frequency. In the quark self energy we have the dressed gluon propagator $D_{\nu\mu}$ and the quark-gluon vertex Γ_ν. The truncation of the quark-gluon vertex together with the resulting form of the quark-DSE is given diagrammatically in Fig.1. The symbol 'YM' denotes contributions from the gluonic sector of QCD whereas the meson exchange diagram contains in principle all possible mesonic fluctuations, see [3] for details. As a first step towards a more complete description, the preliminary results reported here include only pion contributions on a qualitative level.

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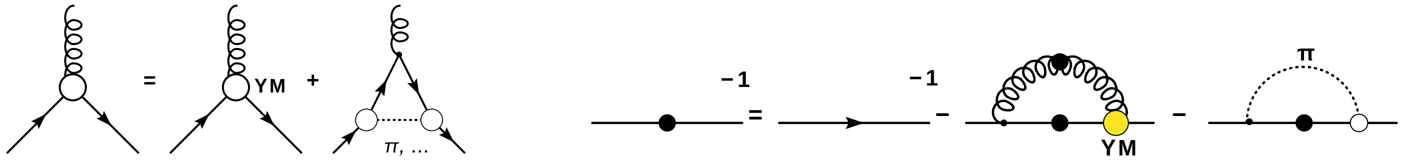

Figure 1: Approximated Dyson-Schwinger equations for the quark-gluon vertex and quark propagator.

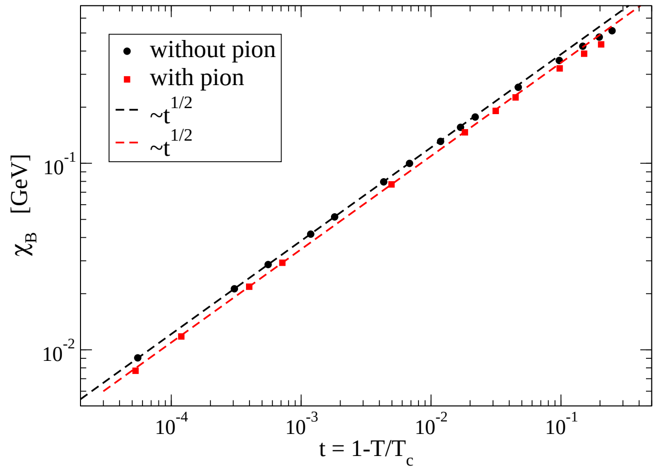

Figure 2: Scaling behaviour of $\chi_B := B(\omega_0, \vec{p}^2 = 0)$ as a function of the reduced temperature.

One possibility to characterise a phase transition is to study order parameters. For a second order phase transition there is a scaling region where the chiral condensate $\langle\bar{\psi}\psi\rangle_\mu$ is proportional to $\sim t^\beta$ with $t = (1 - T/T_c)$ the reduced temperature and β the critical exponent. We found numerically that in the chiral limit $\chi_B := B(\omega_0, \vec{p}^2 = 0)$ equally serves as an order parameter since for $t \sim 0$: $\chi_B \propto \langle\bar{\psi}\psi\rangle_\mu \propto t^\beta$. To extract this behaviour from numerical calculations the critical temperature has to be determined to some accuracy since the scaling sets in not until $t < 0.03$ and the slope in the vicinity of small t is strongly dependent on deviations from T_c. In our (qualitative) study the inclusion of the pion loop decreases the critical temperature by $\sim 5\%$ from $T_c \sim 199$ MeV to $T_c \sim 188$ MeV. The scaling behaviour of χ_B is shown in Fig.2. Both our results with and without the pion backreaction can well be fitted by a power law $\sim t^{1/2}$, which is characteristic of mean field behaviour. (For the vertex dressing without the pion this is in accordance with previous results in the DSE approach summarised in [4].) These results suggest that the contributions necessary to go beyond mean field are not yet included in our approach. So far we do not take care of all temperature dependencies of the pion wave function and decay constants. Furthermore we neglected contributions from the scalar meson channel. A detailed study of these effects is under way and will be reported elsewhere.

References

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